Ranking Invariance between Fuzzy Similarity Measures applied to Image Retrieval

Jean-Francois Omhover
LIP6 – Pole IA
UPMC
8, rue du Cpt. Scott
75015 Paris, France

Marcin Detyniecki
LIP6 – Pole IA
UPMC – CNRS
8, rue du Cpt. Scott
75015 Paris, France

Maria Rifqi
LIP6 – Pole IA
UPMC
8, rue du Cpt. Scott
75015 Paris, France

Bernadette Bouchon-Meunier
LIP6 – Pole IA
UPMC – CNRS
8, rue du Cpt. Scott
75015 Paris, France

***Abstract*— In this paper, we first introduce the fuzzy similarity measures in the context of a CBIR system. This leads to the observation of an invariance in the ranking for different similarity measures. We then propose an explanation to this phenomenon, and a larger theory about order invariance for fuzzy similarity measures. We introduce a definition for equivalence classes based on order conservation between these measures. We then study the consequences of this theory on the evaluation of document retrieval by fuzzy similarity.**

I. Introduction

Almost all of the Image Retrieval systems [2], [8] are based on the same paradigm: one user gives an image as a request, the system compares this image to all of the entries of an image database, and it returns a list of the images that are the most similar to the request. This list order based on the decreasing similarity to the request. The same paradigm is generally used in information retrieval systems.

To compute the similarity between a request and an entry, the system uses its own vector representation, automatically extracted from the images. These vectors summarize the visual information contained in each image and based on them the system can compare pairs of images in this description space. Independanty on the seek of a representative space of description, comparison functions are used to measure the similarity of two pairs of vectors: for instance geometric distances, or similarity measures [6].

To compare binary sets of features, Tversky [11] introduced a formal definition based on a psychological framework. Based on this, an extended definition of similarity measures between gradual sets of features has been derived [1]. These measures are set-theoretic similarity measures, like the classical histogram intersection introduced by Swain and Ballard [9].

The choice of the fuzzy similarity measures is large [5]. All sort of theorical behaviours are possible. Based on this observation we decided to do some experimental tests in order to choose the most appropriate measure. With the image retrieval system we have built [4], we observed the following phenomenon: for any given request, the list of the similar images was always the same for a certain group of the fuzzy similarity measures, independantly on the choice of the features. Of course, the values of the similarities were different, but they were exactly in the same order.

The invariance in ordering has already been a topic of interest of the fuzzy community. For instance, in the field of multicriteria decision, Grabisch [3] studied the invariance for fuzzy connectives. Here, we focus on similarity measures, and we show that equivalence classes exist between them.

In this paper, we first introduce the fuzzy similarity measures in the context of a CBIR system (see section II). This leads to the observation of an invariance in the ranking for different similarity measures. We then propose an explanation to this phenomenon (see section III), and a larger theory about order invariance for fuzzy similarity measures (see section IV). In fact, we introduce a definition for equivalence classes based on order conservation between these measures. In the last section we study the consequences of this theory on the evaluation of document retrieval by fuzzy similarity.

II. Ressemblance Measures for Image Retrieval Systems

Here, we first introduce the definition of fuzzy resemblance measures, then we apply some of these measures to the comparison of global histograms. Finally, we observe an invariance in the ranking provided by different fuzzy similarity measures.

A. Tversky's Contrast Model

Measures of similartity (or dissimilarity) are distinguished into two classes: the geometric one and the set-theoretic one.

Geometric distance models are the most commonly used approach. Objects to be compared are considered as points in a metric space. These models are constrained by 4 properties that a distance has to satisfy: positivity, symmetry, minimality and triangular inequality. Distance models are widely used in the image retrieval field.

These axioms were in particular studied by Tversky [11], who proposed a set-theoretic approach based on more psychological considerations. His study conclude on the very questionable character of the distance axioms. His proposition of a set-theoretic definition of similarity is called Tversky's contrast model. In his scheme, objects to be compared are described by means of sets of binary features. His model considers 5 assumptions (for more details, see [11]) like matching, monotonicity, independence, etc... Let a and b be two objects described respectively by the sets of features A and

B, and s a measure of similarity. The matching assumption tells that $s(a,b) = F(A \cap B, A - B, B - A)$ with F a real function of three arguments. From his model, tversky proposed two mathematical formulations of similarity measures. The first is derived from his representation theorem:

$$S(a,b) = \theta f(A \cap B) - \alpha f(A - B) - \beta f(B - A), \\ \text{for } \theta,\ \alpha,\ \beta \geq 0$$

The second formulation is called the ratio model, that introduces a ratio between common features ($f(A \cap B)$) and different features ($f(B - A)$, $f(A - B)$). The three parameters θ, α and β, are real positive weights that balance the influence of each of these sets:

$$S(a,b) = \frac{\theta f(A \cap B)}{\theta f(A \cap B) + \alpha f(B - A) + \beta f(A - B)}$$

B. Ressemblance Measures

Because of the restriction of the contrast model to binary features, three attempts to generalize the model to fuzzy features have been proposed in literature: [7], [1], [6].

We consider the Bouchon et al. one's [1]. In this framework, for any set Ω of elements, $P_f(\Omega)$ denotes the set of fuzzy subsets of Ω and a fuzzy set measure M is supposed to be given such that $M : P_f(\Omega) \rightarrow \mathbb{R}^+$ and $M(\emptyset) = 0$ and M is monotonous with respect to $\subseteq$. For instance:

- $M_1(A) = \int_\Omega f_A(x)dx$,
- $M_2(A) = \sup_{x \in \Omega} f_A(x)$,
- $M_3(A) = \sum_{count} f_A(x)$ if Ω is countable.

One should notice that this framework also applies to the case of crisp sets.

Definition 1: An *M-measure of comparison* on Ω is a mapping $S : P_f(\Omega) \times P_f(\Omega) \rightarrow [0,1]$ such that $S(A,B) = F_S(M(A \cap B), M(B - A), M(A - B))$,
where F_S is a mapping $F_S : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow [0,1]$ and M a fuzzy set measure on $P_f(\Omega)$.

We denote:

- $X = M(A \cap B)$
- $Y = M(B - A)$
- $Z = M(A - B)$

A measure of comparison captures various families of measures. We focus on those called measures of resemblance.

Definition 2: An *M-measure of resemblance* on Ω is a measure $S(A,B) = F_S(X,Y,Z)$ such that:

- F_S is increasing with X and decreasing with Y and Z
- $F_S(X,0,0) = 1$ for all X
- $F_S(X,Y,Z) = F_S(X,Z,Y)$.

M-measures of resemblance which satisfy an additional property of t-transitivity, for a triangular norm t, are extensions of indistinguishability relations [10], [12] to fuzzy sets. In the case where t is the minimum, we obtain extensions of measures of similarity in the sense of Zadeh.

M-measures of resemblance satisfying the property of exclusiveness:

$$F_S(0,Y,Z) = 0 \quad \text{for all } \ (Y,Z) \neq (0,0)$$

are called *exclusive M-measures of resemblance*.

C. Image Retrieval by Fuzzy Ressemblance

We have developped an image retrieval system, called STRICT ("System for Testing Retrieval by Image ContenT") [4], which is based on a regional representation of the images. Regions are extracted automatically by a segmentation algorithm. For each image of an image database, a description of each of its region is computed and stored. The image retrieval can then be driven on the basis of regional queries. The comparison between the descriptors of these regions is computed by fuzzy M-measures of ressemblance. These measures let us build intuitively modifiable queries, and can be aggregated homogeneously to form composite requests (requests for images containing several regions of interest).

For seek of simplicity, we are going to focus here only on the classical global representation. We represent each image by its global histogram [9], which gives the distribution of pixels' colors for a given color palette $C_1, ..., C_n$. It can be considered as a fuzzy set membership function H_I on the universe $C_1, ..., C_n$. To compute similarities between images, we used fuzzy M-measures of ressemblance to compare their histograms. For two images I_1,I_2, the comparison was done by computing one of the four following measures based on their histograms H_{I_1},H_{I_2}, we denote $X = M(H_{I_1} \cap H_{I_2})$, $Y = M(H_{I_2} - H_{I_1})$, $Z = M(H_{I_1} - H_{I_2})$, M being the area of the given set:

$$S_{jaccard}(X,Y,Z) = \frac{X}{X+Y+Z}$$

$$S_{dice}(X,Y,Z) = \frac{2X}{2X+Y+Z}$$

$$S_{ochiai}(X,Y,Z) = \frac{X}{\sqrt{X+Y}\sqrt{X+Z}}$$

$$S_{Fermi-Dirac}(X,Y,Z) = \frac{F_{FD}(\phi) - F_{FD}(\frac{\pi}{2})}{F_{FD}(0) - F_{FD}(\frac{\pi}{2})}$$

with $F_{FD}(\phi) = \frac{1}{1+\exp\frac{(\phi-\phi_0)}{\Gamma}}$ and $\phi = \arctan(\frac{Y+Z}{X})$, Γ is a positive real and $\phi_0 \in [0, \frac{\pi}{2}]$. The parameter ϕ_0 balances the selectivity of the Fermi-Dirac resemblance, Γ influences its harshness. These four measures are commonly used in fuzzy sets similarity measurement.

We launched several requests on the basis of these different measures, and observed an intruiguishing phenomenon (shown in figure 1): the list and the order of the results was always the same for Jaccard, Dice and Fermi-Dirac measures, little changes were observed with Ochiai's. Obviously, the values returned by each measure were different.

In the following sections, we will study the invariance of the order observed within images ranked by their similarity to a request. For a database of images $I_1, ..., I_n$, the value returned by a measure S for the comparison of each entry to a request R is used to order $I_1, ..., I_n$ by decreasing ressemblance to R.

For every pair of documents I_i, R, each similarity measure as defined in II-B is computed based on three real values X,Y and Z (as denoted in definition 1). Then the problem of ordering pairs of documents by their similarity is is extented to the problem of ordering real triplets of values (X,Y,Z) by a measure of similarity S.

Fig. 1. Two 5-best-results lists using Jaccard and Dice measures applied to image histograms.

This problem is not only significant for the only purpose of global histogram comparison, but also for any fuzzy set representation, or multiple sets of features. For example, in our CBIR system, similarity measures are computed to match regions in each image I_i with regions in a given request R. As we use a best-matching mecanism, the order induced by the similarity measure between pairs of regions in I_i and R significantly influences the result of the matching. It is also significant for the larger field of information retrieval, for instance in text retrieval where similarity measures are also used to compare sets of features.

III. Jaccard, Dice, Fermi-Dirac and Ochiai similarity measures

In this section, we give an explanation of the ranking-invariance observed between Jaccard, Dice and Fermi-Dirac resemblance measures. We also show that this invariance is not always observed, for example between Jaccard and Ochiai resemblance measure. The properties studied in this case provide a starting point for the generalisation proposed in the next sections.

A. Equivalence of Jaccard and Dice

From a trivial analysis of the expression of Jaccard and Dice measures, we can write Dice's measure as a function of Jaccard's measure, and vice versa. For any given (X, Y, Z) as expressed in II-B:

$$S_{dice}(X, Y, Z) = \frac{2X}{2X+Y+Z} = f(S_{jaccard}(X, Y, Z))$$
$$S_{jaccard}(X, Y, Z) = \frac{X}{X+Y+Z} = f^{-1}(S_{dice}(X, Y, Z))$$

$$\text{with } f : \begin{cases} [0,1] \to [0,1] \\ x \mapsto \frac{2x}{1+x} \end{cases} \quad \text{and } f^{-1} : \begin{cases} [0,1] \to [0,1] \\ x \mapsto \frac{x}{2-x} \end{cases}$$

Furthermore, f and f^{-1} are strictly increasing functions. Then, for any (X, Y, Z), (X', Y', Z'), calculated from two pairs of fuzzy sets,

if $\quad S_{jaccard}(X, Y, Z) \leq S_{jaccard}(X', Y', Z')$

then by applying f, which is increasing
$$S_{dice}(X, Y, Z) \leq S_{dice}(X', Y', Z')$$

and then, by applying f^{-1} which is also increasing,
$$S_{jaccard}(X, Y, Z) \leq S_{jaccard}(X', Y', Z')$$

From above, we can conclude that for any given request image R, and any set of images $L = \{I_1, ..., I_n\}$, if we use Jaccard's measure to order L by the decreasing similarity of its elements to R, then using Dice's measure instead will order L exactly the same way.

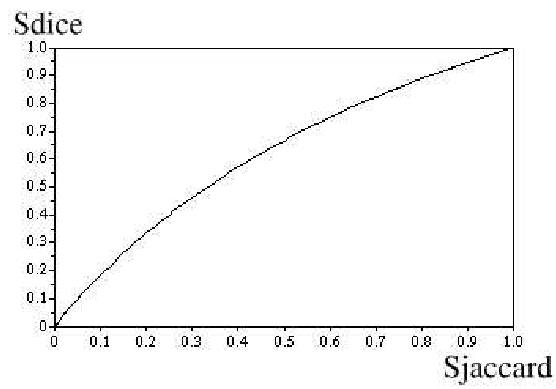

Fig. 2. Plot of the set $\{(S_{jaccard}(O), S_{dice}(O))/O = (X, Y, Z) \in \mathbf{R}^{+3}\}$

On the plot presented figure 2, we can observe this property in another representation. On this figure, we have plot in $[0, 1]^2$ the set C of the couples of values that Jaccard and Dice take on any (X, Y, Z). We notice that this set is in fact the graph of the function f cited above. As this function is strictly increasing, if we take any two couples of values in C, they are found in an increasing configuration (the slope of the straight line passing by these two points is strictly positive). Then we can never found any order change between two couples of Jaccard and Dice values.

B. Equivalence of Jaccard and Fermi-Dirac

The same analysis can be driven between Fermi-Dirac and Jaccard resemblance measures. From the expression of Fermi-Dirac's measure:

$$S_{FD}(X, Y, Z) = \frac{F_{FD}(\phi) - F_{FD}(\frac{\pi}{2})}{F_{FD}(0) - F_{FD}(\frac{\pi}{2})}$$
$$\text{with } F_{FD}(\phi) = \frac{1}{1+\exp\frac{(\phi-\phi_0)}{\Gamma}}$$

where $\phi = \arctan(\frac{Y+Z}{X})$, Γ is a positive real and $\phi_0 \in [0, \frac{\pi}{2}]$. The central computation of this resemblance measure

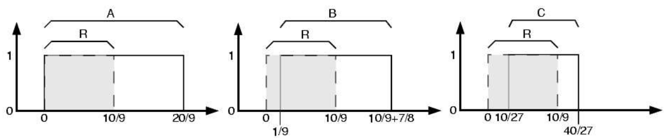

Fig. 3. Four subsets R,A,B,C of $\boldsymbol{R}$ defined by their [crisp] membership functions

lies within the "angle" ϕ, which equals zero when the fuzzy sets compared are the same, and tends to $\frac{\pi}{2}$ when they become different. But ϕ can be written as a function of Jaccard's similarity measure: that is $\phi = f(S_{jaccard}(X,Y,Z))$

$$\text{with } f : \begin{cases} [0,1] \rightarrow & [0, \frac{\pi}{2}] \\ x \mapsto & \arctan(\frac{x-1}{x}) \text{ if } x \in]0,1] \\ & \frac{\pi}{2} \text{ if } x = 0 \end{cases}$$

f is strictly decreasing with x, which means that ϕ is decreasing with the value of $S_{jaccard}(X,Y,Z)$. F_{FD} is strictly decreasing with ϕ. Then S_{FD}, which is a composition of these two functions, is strictly increasing with $S_{jaccard}(X,Y,Z)$. This composition leads to the formulation of an increasing bijection between S_{FD} and $S_{jaccard}$ values. This leads to the same conclusion as in section III-A: to order a set of images by their decreasing similarity to any given request, we can blindly use Jaccard or Fermi-Dirac measure.

C. Non-Equivalence of Jaccard and Ochiai

To show that the property of ranking conservation is not always observed, we focus on Jaccard and Ochiai measures. Our approach is to exhibit an example of ranking that is not conserved. This example is discussed here between three hand-made sets R, A and B, as displayed in figure 3. On the one hand, we measure the similarity first between R and A, then between R and B using the Jaccard's measure. On the other hand, we measure the same similarities with the Ochiai's measure. As defined in II-B, we have to compute the areas:

$$\begin{cases} X_1 = M(R \cap A) = \frac{10}{9} \\ Y_1 = M(A - R) = \frac{10}{9} \\ Z_1 = M(R - A) = 0 \end{cases} \begin{cases} X_2 = M(R \cap B) = 1 \\ Y_2 = M(B - R) = \frac{7}{8} \\ Z_2 = M(R - B) = \frac{1}{9} \end{cases}$$

which leads to the following values of similarity:

$$\begin{cases} S_{jaccard}(X_1,Y_1,Z_1) = 0.5 \\ S_{ochiai}(X_1,Y_1,Z_1) = 0.707 \end{cases}$$

$$\begin{cases} S_{jaccard}(X_2,Y_2,Z_2) = 0.503 \\ S_{ochiai}(X_2,Y_2,Z_2) = 0.692 \end{cases}$$

which means that $S_{jaccard}(R,A) < S_{jaccard}(R,B)$ but $S_{ochiai}(R,A) > S_{ochiai}(R,B)$. The order between A and B due to their similarity to R is not conserved from jaccard to ochiai. Furthermore, we are about to show that Ochiai can't be written as a function of Jaccard's measure. Let's consider the set C as shown in figure 3, and the following areas:

$$\begin{cases} X_3 = M(R \cap C) = \frac{20}{27} \\ Y_3 = M(C - R) = \frac{10}{27} \\ Z_3 = M(R - C) = \frac{10}{27} \end{cases}$$

If we evaluate Jaccard and Ochiai measures' values for the comparison of R and C, we obtain the following values:

$$\begin{cases} S_{jaccard}(X_3,Y_3,Z_3) = 0.5 \\ S_{ochiai}(X_3,Y_3,Z_3) = 0.66 \end{cases}$$

which means that $S_{jaccard}(R,A) = S_{jaccard}(R,C)$ but $S_{ochiai}(R,A) \neq S_{ochiai}(R,C)$. This makes it impossible for Ochiai's measure to be written as a one-valued function of Jaccard's. This assertion is also justified by the observation of figure 4.

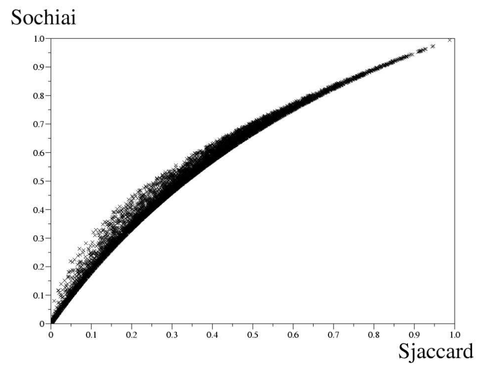

Fig. 4. Plot of the set $C = \{(S_{jaccard}(O), S_{ochiai}(O))/\forall O \in \boldsymbol{R}^{+3}\}$

Here, we have roughly plot the set C of the couples of values that Jaccard and Dice take on any $O = (X,Y,Z)$. We notice that this set can not be reduced to a function, and furthermore, there is enough space in the band to choose two points such as the line passing on them is decreasing. This will mean that the order between the couple of images that these two points represent will change. Another way to see it is that to a single value of similarity issued from Jaccard's measure, correspond many possible values of Ochiai's, and vice versa.

IV. Classes of Equivalent Similarity Measures

Based on the behaviour observed for the four measures presented in section III, we propose three definitions of equivalence relations between similarity measures based on order conservation. We show that these three definitions lead to the same equivalence class.

A. Equivalence in order

The first definition takes its root in the most basic conclusion of section III: between certain measures, the order of the values issued from comparison between elements is conserved, between some others, this order may change. We then propose to study the following relation:

Definition 3: For any similarity measures S_a and S_b, we say S_a is "equivalent in order" to S_b if and only if

$$\forall O = (X,Y,Z) \in \boldsymbol{R}^{+3}, \forall O' = (X',Y',Z') \in \boldsymbol{R}^{+3}$$
$$S_a(O) \leq S_a(O') \iff S_b(O) \leq S_b(O')$$

This relation is trivially reflexive, symetric and transitive. Then it defines equivalence classes within similarity measures.

In section III, we showed that order is invariant between Jaccard, Dice and Fermi-Dirac resemblance measures: these three are in the same equivalence classes in the sense of definition 3. We also showed that the order changes between Jaccard and Ochiai, these two measures then belong to different classes of equivalence.

B. Equivalence by a function

The second definition relies on the fact that, in section III, between the measures that produce the same order between values of comparisons, we were able to write one as an increasing function of the other one. For example, we have shown this property between Jaccard and Dice similarity measures (see III-A).

Definition 4: For similarity measures S_a and S_b, we say S_a is "equivalent by a function" to S_b if and only if there exists a function:

$$f : \begin{cases} Im(S_a) \to Im(S_b) \\ x \mapsto f(x) \end{cases} \text{ strictly increasing,}$$
$$\text{such as } S_b = f \circ S_a$$

with $Im(S_a) = \{\alpha / \exists (X, Y, Z) \in \mathbb{R}^{+3}, \alpha = S_a(X, Y, Z)\}$. By definition, any f as in definition 4 is surjective. By its strictly increasing property, it is also injective. Then, this f is a bijection between $Im(S_a)$ and $Im(S_b)$. The relation defined above is then symetric. It is also trivially reflexive and transitive. Then, it also defines a relation of equivalence for similarity measures.

In section III, we showed that Jaccard, Dice and Fermi-Dirac were linked one to the other by a function. Therefor these three measures are in the same equivalence classes in the sense of definition 4. We also showed that Ochiai's measure could not be written as a function of Jaccard's, in the sense of 4, they belong to different equivalence classes.

Here, we can notice that if two S_a and S_b are equivalent by a function, their sets of values $Im(S_a)$ and $Im(S_b)$ can be mapped on each other. If two measures have unmappable sets of values, for example if one is continuous (and has its values in $[0, 1]$), and the other is discrete (and has values in $\{0, 0.2, 0.4, 0.6, 0.8, 1\}$), then it is impossible for them to be equivalent by a function.

The two definitions 3 and 4 are different in their formulation, but we have shown that this difference is only virtual. These two definitions lead to the same classes of equivalence.

Theorem 1: Definition 3 and 4 are equivalent. They define the same equivalence classes.

If two measures induce the same order, we can build a function f that is strictly increasing so that one can be written as a composition of the other with f. Inversely, if two measures are related by a strictly increasing function, then we can show that they induce the same order. In other words, "equivalence in order" leads to "equivalence by a function", and vice versa.

C. Equivalence in level-sets for continuous measures

We call level-set of a similarity measure S_i at the height λ the set noted S_i^λ defined by:

$$S_i^\lambda = \{(X, Y, Z) / S_i(X, Y, Z) = \lambda\}$$

If two measures of resemblance S_a and S_b are equivalent by a function (or by order), then the following proposition can be shown:

$$\forall O = (X, Y, Z), O' = (X', Y', Z'),$$
$$S_a(O) = S_a(O') \iff S_b(O) = S_b(O')$$

This proposition implies that S_b takes one and only one value on any level set of S_a, and vice versa. Based on this, we can propose the following definition:

Definition 5: For any similarity measures S_a and S_b, S_a is said "equivalent in level-sets" to S_b if:
$\forall \beta \in Im(S_b), \exists! \alpha$(unique) in$Im(S_a)$ so that $S_a^\alpha = S_b^\beta$

This relation is trivially reflexive. It can easily be shown that it is also symmetric and transitive. To link this definition to the two definitions of equivalence proposed above, we need to introduce the following theorem:

Theorem 2: For any continuous similarity measures S_a and S_b,
If $\exists f : \begin{cases} Im(S_a) \to Im(S_b) \\ x \mapsto f(x) \end{cases}$ such as $S_b = f \circ S_a$
then f is monotonous, not decreasing.

If two continuous similarity measures S_a and S_b are equivalent in level-sets, we can build a function that links S_a to S_b by composition. From theorem 2, this function is monotonous and not decreasing. It can easily be shown that it is indeed strictly increasing by using the reflexivity of relation 5. Reversely, if two similarity measure S_a and S_b are equivalent by a function, it can be shown that it is equivalent by level-sets. From this, we obtain the second equivalence theorem.

Theorem 3: Definition 3, 4 and 5 are equivalent for continuous similarity measures.

When dealing with continuous measures, equivalence can be demonstrated by using theorem 3 and definition 5: if the level-sets are matching one-to-one, the measures are equivalent.

We have used this to demonstrate various other theorems. For example, we have pointed out the equivalence classes within the ressemblance measures proposed by Tversky in his ratio model: two Tversky's ratio-model-measures are equivalent if they identically weight the influence of the difference sets, in other words if the ratio α/β is the same in the two measures (see II-A).

V. Consequences of using equivalent measures to order pairs of documents

This section studies the consequences of using measures of a same given equivalence class, in other words, the implications of order invariance to document retrieval in general.

The first direct consequence lies in the result the user will observe. In fact, he will obtain exactly the same results ranking for any of the measures belonging to a given equivalence class.

Second, these results do not only apply to a global description of an image. As pointed out in section II-C, if a best-matching procedure uses comparisons of pairs of objects (in our case, regions), the result will be dependant only on the order of the resemblance values. If that order is conserved from a measure to another, the final matching will be identical.

The first consequence is more fundamental and concerns the evaluation of the retrieval. For two equivalent measures, any recall/precision comparison based on the lists of the entries retrieved will conclude to the exact same accuracy. In particular, if order is invariant between two lists of results, the question of comparing the numbers of pertinent documents in the first 10 results is void.

It seems that when using similarity measures of a same equivalence class, only the order (and not the value itself) is important. But this is true only if there is not a post-processing procedure which may destroy the order-conservation. For instance the aggregation of several similarities gives a sense to the values it self. Another example may be if a decision has to be taken on the degree of the similarity of two objects, a greater or lower value or the similarity will induce a stronger or weaker decision. In this context, works on the selectivity power will find all their interest.

VI. Conclusion

From the observation of an invariance in the order of the results retrieved by a CBIR system based on fuzzy resemblance measures, that we have described and explained, we have proposed three definitions of equivalence that are based on order invariance. As we have shown, these different definitions lead to the same equivalence classes. This theoretical framework let us demonstrate various properties, and implies many consequences regarding the order of the values issued from a comparison between pairs of objects used in some applications.

References

[1] B. Bouchon-Meunier, M. Rifqi, and S. Bothorel. Towards general measures of comparison of objects. *Fuzzy Sets and Systems*, 84(2):143–153, 1996.

[2] M.D. Flickner, H. Sawhney, W. Niblack, J. Ashley, Q. Huang, B. Dom, M. Gorkani, J. Hafner, D. Lee, D. Petkovic, D. Steele, and P. Yanker. Query by image and video content: The qbic system. *Computer*, 28(9):23–32, September 1995.

[3] M. Grabisch. On equivalence classes of fuzzy connectives: The case of fuzzy integrals. *IEEE Transactions on Fuzzy Systems*, 8(1):96–109, 1995.

[4] J.F. Omhover, M. Detyniecki, and B. Bouchon-Meunier. A region-similarity-based image retrieval system. In *IPMU'04*, Perugia, Italy, 2004 (submitted).

[5] M. Rifqi, M. Detyniecki, and B. Bouchon-Meunier. Discrimination power of measures of resemblance. In *IFSA'03*, Istanbul, Turkey, 2003.

[6] S. Santini and R. Jain. Similarity measures. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 21(9), September 1999.

[7] K. Shiina. A fuzzy-set-theoretic feature model and its application to asymmetric data analysis. *Japanese psychological research*, 30(3):95–104, 1988.

[8] J.R. Smith and S.F. Chang. Visualseek: a fully automated content-based image query system. In *ACM Multimedia '96*, November, 1996.

[9] M.J. Swain and D.H. Ballard. Color indexing. *International Journal of Computer Vision*, 7(1):11–32, 1991.

[10] E. Trillas and L. Valverde. On implication and indistinguishability in the setting of fuzzy logic. In J. Kacprzyk and R. R. Yager, editors, *Management Decision Support Systems Using Fuzzy Sets and Possibility Theory*. Verlag TUV, Rheinland, 1984.

[11] A. Tversky. Features of similarity. *Psychological Review*, 84:327–352, 1977.

[12] L. Valverde. On the structure of t-indistinguishability operators. *Fuzzy Sets and Systems*, 17:313–328, 1985.