M. Detyniecki and R. R. Yager, "Ranking fuzzy numbers using alpha-weighted valuations,"
*International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*,
vol. 8 (5), pp. 573-592, 2001.

# RANKING FUZZY NUMBERS USING α-WEIGHTED VALUATIONS

MARCIN DETYNIECKI
*LIP6 - CNRS -University of Paris VI*
*4, place Jussieu*
*75005 Paris, France*
Marcin.Detyniecki@lip6.fr

RONALD R. YAGER
*Machine Intelligence Institute*
*Iona College*
*New Rochelle, New York 10801, USA*
Yager@panix.com

We studied here on some simple examples the interaction between valuation family, parameters and ranking result. The ranking method studied is based upon the idea of associating with a fuzzy number a scalar value, its valuation, and using this valuation to compare and order fuzzy numbers. The valuation method considered was introduced initially by the Yager and Filev. This valuation consists in the integration over α-levels, of the average of each α-cut weighted by a weight distribution function. We finish by introducing a new weight distribution function.

## 1. Introduction

In many applications of fuzzy set theory to decision making we are faced with the problem of selecting one from a collection of possible solutions, and in general we want to know which is the best one. This selection process may require that we rank or order fuzzy numbers. While it is clear how to order scalar numbers, the ordering problem with respect to fuzzy numbers is not always obvious. From the beginning of the development of the fuzzy set theory [1] the problem of comparing fuzzy subsets was studied and appeared to be an important and difficult problem. See [2,3] for a review of some methods. We focus here on one particular ranking method introduced by Yager [4]. This ranking method is based upon the idea of associating with a fuzzy number a scalar value, its valuation, and using this valuation to compare and order the fuzzy numbers. The originality of this method is the valuation process. Later Yager and Filev [5] improved this valuation method founding their work on expected value type valuations, which are based upon the transformation of fuzzy subset into an associated probability distribution. They developed in [5] a number of families of parameterized valuation functions.

In this paper we concentrate our attention on the ranking process based on the use of these particular valuations methods. We will try to understand, using simple examples, the interaction between valuation family, parameters and ranking result. We will begin in the first part by introducing the valuation method. Then we will see how to compute the valuation of a trapezoidal fuzzy number. We will analyze and interpret the results to obtain a deeper understanding of the process. In the second part we will see how different families can give different orderings for the same collection of triangular fuzzy numbers. We will see that inside a parameterized family we can invert the ordering of two fuzzy numbers by changing the value of the parameter. After giving a general interpretation of the valuation method we show the inherent constraints of the weighting functions proposed by Yager and Filev. We propose a way to relax these limitations. By doing this, we discover two complementary families, which gives us a group of four different weighting functions. We study their behavior and we give an interpretation. Based on this reading we explain the advantage of using this intuitive group of weighting functions in the framework of ranking process and more generally as a defuzzification method. This leads us to conclude by giving a comparison with other standard defuzzification methods.

## 2. Valuation method

One general approach to the problem of comparison of fuzzy numbers is to associate with a fuzzy number $F$ some representative value, $Val(F)$, and to compare the fuzzy subsets using these single representative values. An example of an approach in this spirit was introduced by Yager [4]. He suggested to use:

$$Val(F) = \int_0^1 Average\left(F_\alpha\right) \cdot d\alpha \tag{1}$$

where $F_\alpha = \{x \mid F(x) \geq \alpha\}$ is the α-level set of $F$. We note that if $F$ is not normal, we integrate between 0 and $\max_x(F(x))$ and multiple by $1/\max_x(F(x))$.

Yager and Filev [5], basing their work upon the transformation of a fuzzy subset into an associated probability distribution [6], extended this formulation and developed a generalized formulation for a class of valuation functions

$$Val(F) = \frac{\int_0^1 Average\left(F_\alpha\right) \cdot f(\alpha) \cdot d\alpha}{\int_0^1 f(\alpha) \cdot d\alpha} \tag{2}$$

In the above $f$ is a mapping from [0,1] to [0,1].

In [5] Yager and Filev proposed two complementary parameterized functions from this class of valuation functions. One is an increasing family of functions and the other one is decreasing. Let us take a look at these two families:

### 2.1. *The increasing family*

$$f:\alpha \to f(\alpha)=\alpha^{q} \quad \text{with} \quad q \geq 0 \tag{3}$$

We have two interesting *particular cases* (the extremes):

- for $q = 0$ we obtain $f$ constant equal to 1. So, the valuation is given by the original equation (1)
- for $q \to \infty$ we obtain the Dirac function translated to 1. So, the valuation is $Val(F) = Average(F_1)$. It is the average of the core.

In this family we notice that we are placing more emphasis on the higher α-level sets, and that the larger $q$ the more emphasis we give to the higher level sets. In the limiting case, when α approaches infinity we just use $F_1$ (the core of F)

### 2.2. *The decreasing family*

It is the complementary case of the increasing family:

$$f:\alpha \to f(\alpha)=(1-\alpha)^{q} \quad \text{with} \quad q \geq 0 \tag{4}$$

Here we also have two interesting *particular cases* (the extremes):

- for $q = 0$ we obtain $f$ constant equal to 1, that gives the valuation expressed in the original equation (1)
- for $q \to \infty$ we obtain the Dirac function. So, the valuation is $Val(F) = Average(F_0)$. It is the average of the support.

In the decreasing family for $q > 0$, we notice that we are placing more emphasis on the lower α-level sets, and that the larger $q$ the more emphasis we give.

At this point we have presented the general valuation method (2) introduced by Yager and Filev [5] and two parameterized valuation families that are particular cases of (2). In order to understand the valuation process, mathematically defined before; let us now take a precise look at an example.

## 3. Studying on Examples

In order to focus our attention on the valuation process we are going to use one of the simplest type fuzzy set (but at the same time quite general): the trapezoidal fuzzy numbers.

### 3.1. *Computing the Valuation of Trapezoidal Fuzzy Number*

#### 3.1.1. *Notations*

We are going to describe a trapezoidal fuzzy set by: $T$ (left support, left core, right core, right support). In this way, the fuzzy set represented in the figure will be noted: $T$ (1,6,8,9).

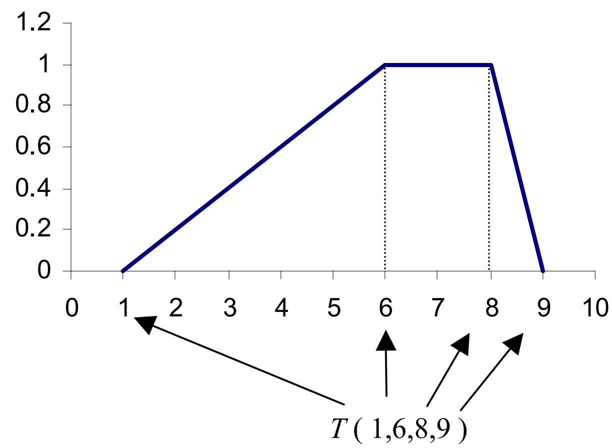



Fig. 1. Notation for a trapezoidal fuzzy set

And the trapezoidal fuzzy subset $T(a, b, c, d)$ will have the membership function:

$$T(x) = \begin{cases} 0 & for \quad x < a \\ \dfrac{x-a}{b-a} & for \quad a \leq x \leq b \\ 1 & for \quad b \leq x \leq c \\ \dfrac{d-x}{d-c} & for \quad c \leq x \leq d \\ 0 & for \quad x > d \end{cases} \tag{5}$$

3.1.2. *Computing the valuation for every f*

If we want to compute the valuation proposed by the formula (2), we need to compute $Average\,(T_\alpha)$. For the case of the trapezoidal this is very simple:

$$Average\,(T_\alpha) = \frac{u_\alpha + v_\alpha}{2} \tag{6}$$

We can obtain $u_\alpha$ and $v_\alpha$ with the help of the membership functions (5):

$$u_\alpha = (b-a)\cdot\alpha + a \quad \text{and} \quad v_\alpha = d - (d-c)\cdot\alpha \tag{7}$$

Then the valuation formula (2) becomes:

$$Val(T) = \frac{\frac{1}{2}\int_0^1 \left[(b+c)\cdot\alpha + (1-\alpha)\cdot(a+d)\right]\cdot f(\alpha)\cdot d\alpha}{\int_0^1 f(\alpha)\cdot d\alpha} \tag{8}$$

We can put this equation on to the following form:

$$Val\left(T(a,b,c,d)\right) = \left(\frac{b+c}{2}\cdot w\right) + \left(\frac{a+d}{2}\cdot(1-w)\right) \tag{9}$$

Where w is computed by:

$$w = \frac{\int_0^1 \alpha\cdot f(\alpha)\cdot d\alpha}{\int_0^1 f(\alpha)\cdot d\alpha} \tag{10}$$

The first thing we observe, in formula (9), is that this valuation is a weighted-mean of the average of the core and the average of the support. One evident result we obtain then is that the valuation (2) for any function f will be between the middle point (average) of the core and the middle point (average) of the support (because w$\in[0,1]$).

3.1.3. *Computing the valuation for Yager and Filev $f$ functions*

Let us now compute the actual valuation for a trapezoidal fuzzy set, using the parameterized function proposed by Yager and Filev [5]:

- for the increasing case ( $f(\alpha)=\alpha^q$ ) we have $w=\frac{q+1}{q+2}$. So we obtain for $q$=0 the average between the middle point of the core and the middle point of the support. If $q$ increases the valuation will move from this initial value until its limit (for $q$=∞) where the valuation will be the middle point of the core.
- for the decreasing case ( $f(\alpha)=(1-\alpha)^q$ ) we have $w=\frac{1}{q+2}$. So we obtain for $q$=0 the average between the middle point of the core and the middle point of the support. If $q$ increases the valuation will move from this initial value until its limit (for $q$ = ∞) where the valuation, this time, will be the middle point of the support.

We can summarize this on a graph:

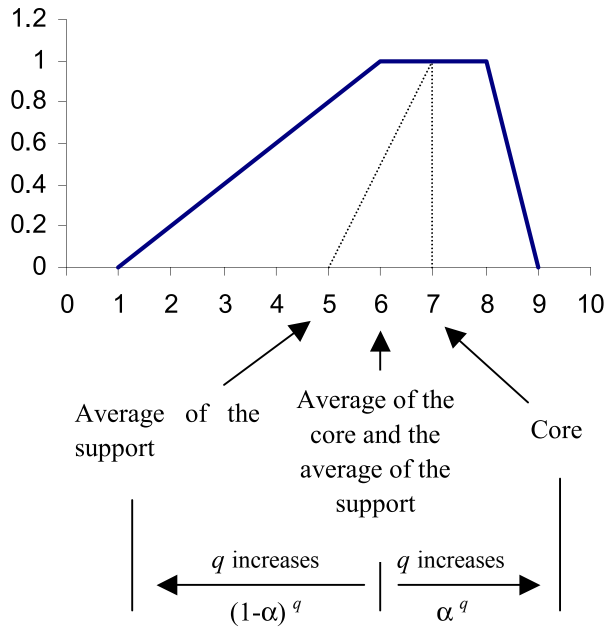



Fig. 2. Variability of the Valuation for Yager and Filev $f$ functions

At this point we have only studied the valuation process, let us now take a look at the interaction between valuation method and ranking. More precisely we are going to analyze on an example the consequences of using this or that valuation family.

### 3.2. *Understanding Ranking with triangular fuzzy Number*

In order to focus our attention only on the ranking as consequence of a particular valuation process we are going to use one of the simplest type fuzzy set: the triangular fuzzy numbers.

#### 3.2.1. *Notations*

A triangular fuzzy number is a particular case of trapezoidal fuzzy number in which the core is reduced to a point. To facilitate the notations we are going to note the triangular fuzzy number $T(l,c,c,r)$ simply by $T(l,c,r)$.

3.2.2. *The importance of f on the ranking*

To understand the interest of the introduction of the function $f$ let us consider the following collection triangular fuzzy sets: $T_a$ ( 5, 6, 13 ) ; $T_b$ ( 3, 7, 13 ) ; $T_c$ ( 5, 8, 9 ) ; $T_d$ ( 2, 9, 10 ) .

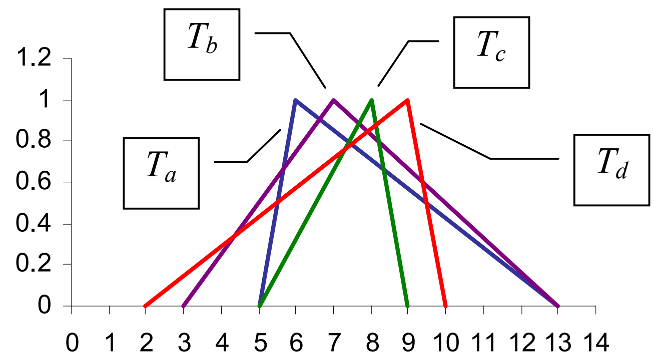



Fig. 3. Example of a group of triangular fuzzy sets that will be ranged

We observe that there is a great variety, but if we use the valuation formula (1), we will obtain that all these fuzzy numbers have the same valuation (7.5). So, we cannot make an ordering. The explanation of this is that low α-levels are compensated with the high α-levels. Let us now take a look at what happens if we use the function *f*.

3.2.3. *Using Yager and Filev f functions*

If we now use the formula (2) with the Yager and Filev increasing $f$ (3), the valuation will give for any $q \neq 0$ the following order: $T_a < T_b < T_c < T_d$. And in the extreme case ($q = \infty$) the valuations will be 6,7,8,9 respectively. We obtain in this way an ordering.
But if we use now the formula (2) with the Yager and Filev decreasing $f$ (4), then the valuation will give for any $q \neq 0$ the opposite ordering: $T_a > T_b > T_c > T_d$. With the valuations 9,8,7,6 respectively for the extreme case ($q = \infty$). We obtain here once again an ordering, but it is the opposite one.
We may be surprised by the fact that we obtain two opposite orderings. But we should not forget that we used two different families to do the valuations. We recall here that the increasing family defined in (3) emphasizes the higher α levels, while the decreasing family defined in (4) emphasizes the lower α levels, and that is why we obtain two different rankings.
Summarizing we had a collection of very different fuzzy triangular numbers, and we were not able to order them with the classical approach. We observed then that for the two different families we obtained two different orderings. If we take a closer look we note that the ordering was the same inside each family. We will see in the next paragraph, with another example, that this is not always the case.

### 3.3. *The influence of q*

We have seen that depending on the family we have chosen we can obtain different orderings. In this section, we will show that the ordering can be changed inside the same family only by modifying the parameter $q$ (the emphasis). To see this we will use examples of pairs of triangular fuzzy numbers.

Let us first analyze the case of the ***increasing family*** defined in (3). For this particular case let us study the following triangular fuzzy sets: $T_a$ ( 1, 7 , 9) and $T_b$ ( 4, 6 , 12).

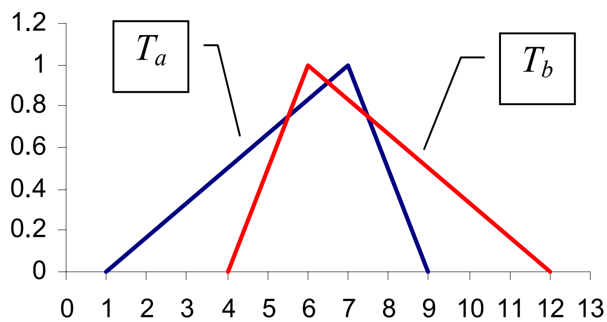



Fig. 4. Two triangular fuzzy sets that give different ranking for different value of $q$

Let us compare on a table the valuation values of each fuzzy number for $q \geq 0$ :

| $q$ | Val ($T_a$) | Val ($T_b$) | Comparison |
|---|---|---|---|
| 0 | 6 | 7 | < |
| 1 | 6.33 | 6.66 | < |
| 2 | 6.5 | 6.5 | = |
| 3 | 6.6 | 6.4 | > |
| ∞ | 7 | 6 | > |

Table 1. Valuation values of each fuzzy number for $q \geq 0$

We observe that for $0 \leq q < 2$ we have $T_a < T_b$, for $q = 2$ $T_a = T_b$ and for $q > 2$ $T_a > T_b$ . More generally we can compare the variation of the valuation using $q$ as a parameter (see Fig. 5).

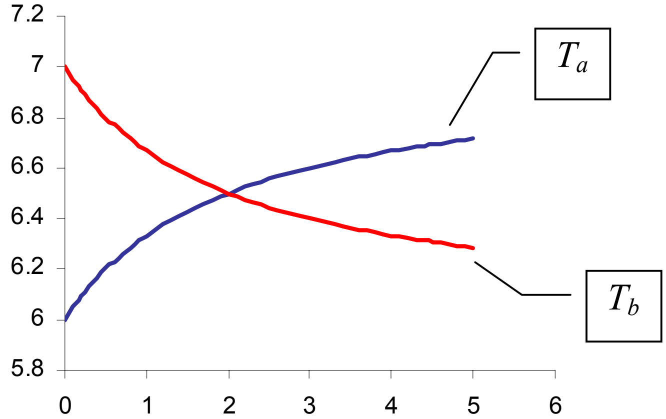



Fig. 5. Value of the valuation for the two triangular fuzzy sets in function of $q$

We have an analogous example for the ***decreasing family*** defined in (4). Let us consider now the fuzzy sets: $T_a$ ( 1, 7 , 9) and $T_b$ ( 2, 4 , 10).

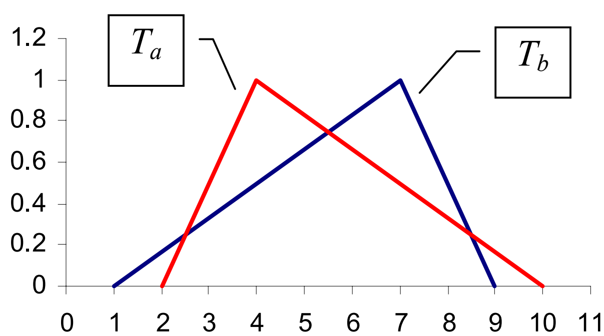



Fig. 6. Two triangular fuzzy set that give different ranking for different value of $q$

Here we represent once again on a graph the valuation $q$ being the variable, and we notice once again that for $q > 2$ the ordering is inverted.

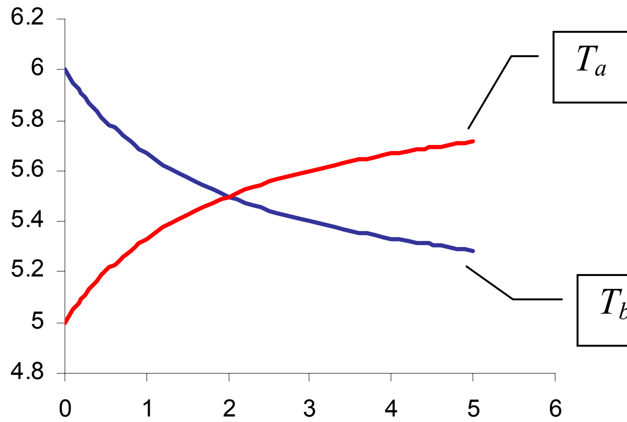



Fig. 7. Value of the valuation for the two triangular fuzzy sets in function of $q$

We notice with these two examples that the ordering can change if we change the emphasis we give to the low or to the high α-levels. Let us now give a general interpretation of the action of the function *f*.

**4. Interpretation**

If we focus our attention on the valuation formula (2) we see that actually $f(\alpha)$ is a weight that we put on the average of the α-cut. In other words, if for $\alpha \in [0,1]$ the value of $f$ is high, then it means that we give a heavy weight to this α-level. Having this in mind let us draw on the same graph the increasing $f$ function (3) for different values of $q$ (the variable being α):

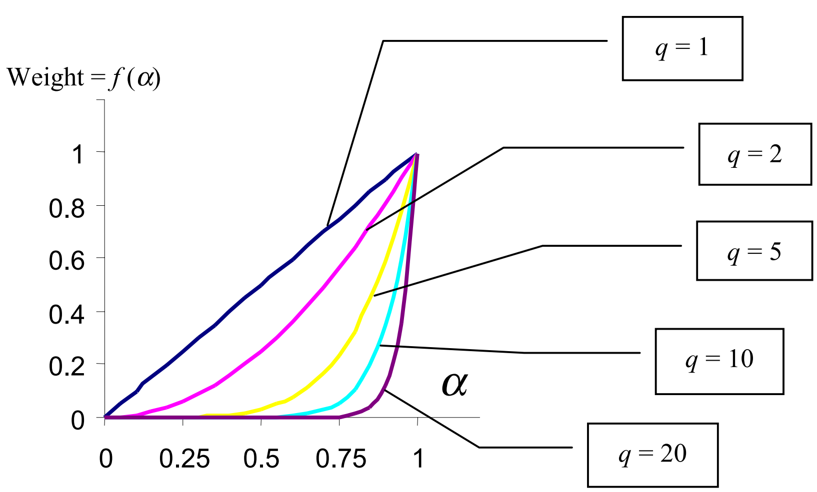



Fig. 8. Yager and Filev increasing weight function $f$ for different values of $q$

We observe here that the increase of $q$ makes the function $f$ flatter at the beginning, and more tilted at the end. The increase of $q$ actually reduces the weight of the low α-levels much more than for the high α-levels. In this way the distribution of weights changes so that the high α-levels increases their importance relatively to the low α-levels.
We observe an analogous behavior for the complementary case: the decreasing family (4):

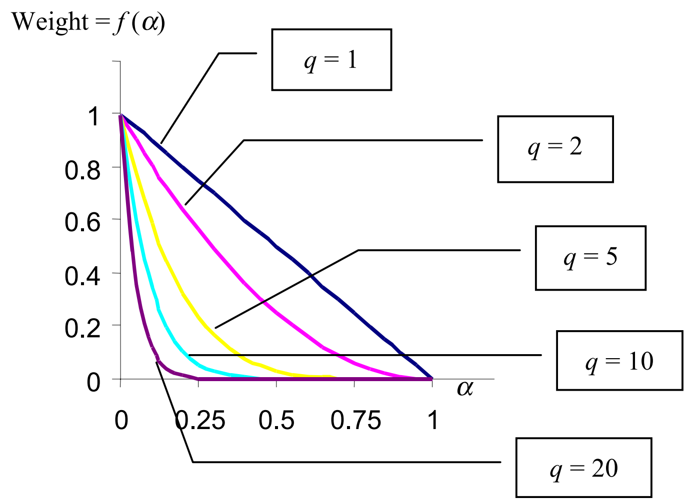



Fig. 9. Yager and Filev decreasing weight function $f$ for different values of $q$

We observe that the increase of $q$ makes the function $f$ flatter at the end, and more tilted at the end. The increase of $q$ actually reduces the weight of the high α-levels much more than for the low α-levels. In this way the distribution of weights changes so that the low α-levels increases their importance relatively to the high α-levels.
What we explained just before is what actually Yager and Filev meant by 'emphasis'. If we take a look at the graph and at formula (3), we discover that we have two fixed points for each family at α=0 and at α=1. This implies that for the increasing family $f(0)=0$ and $f(1)=1$ for all $q$, while for the decreasing family $f(0)=1$ and $f(1)=0$. In the following we shall consider extensions, which allow more freedom in selecting the weights assignment to the core and to the support.

**5. Definition of functions pro-core and pro-support**

We saw in the last paragraph that the increasing function $\alpha^q$ and decreasing function $(1-\alpha)^q$, assigned the weights to the core and to the support to either zero or one. There are several ways to relax this constraint. We choose to use a linear transformation that assigns the weights of the core and support to the desired values and that does not change them with the variation of $q$.

Let $A_i$ be the weight assigned to the core and $B_i$ the weight assigned to the support. We propose to use:

$$f_S(\alpha) = A_S + (B_S - A_S)\cdot \alpha^q \qquad \text{with } q \geq 0 \tag{11}$$

$$f_C(\alpha) = B_C + (A_C - B_C)\cdot (1-\alpha)^q \qquad \text{with } q \geq 0 \tag{12}$$

We will call the family of functions (11) the pro-support and the family (12) the pro-core. In order to understand the behavior of these new families, we compare the pro-support and the pro-core functions having the same core-weight ($B_i$) and support-weight ($A_i$). See Fig. 10 and Fig. 11.

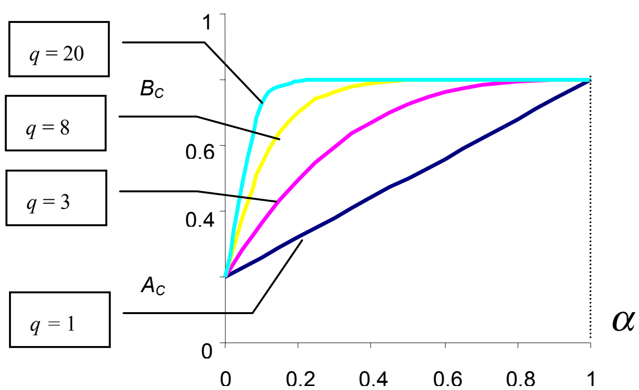



Fig. 10. Functions pro-core with $A_C = 0.2$ , $B_C = 0.8$ and different values of $q$

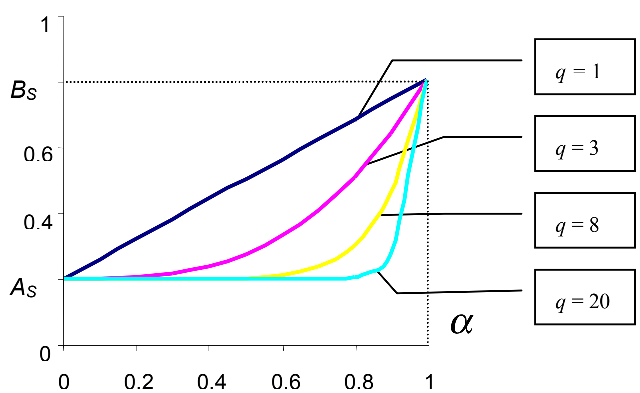



Fig. 11. Functions pro-support with $A_S = 0.2$ , $B_S = 0.8$ and different values of $q$

For the pro-core function (Fig. 12), we observe that when the value of $q$ increases then the weights tend to the weight of the core. In an analogous way for the pro-support (figure#13) the weights will tend to the weight of the support. This tendency inspired the names of these two families. We can express this property in a mathematical way:

- For the pro-support family, we have:

$$\text{For } \alpha \neq 0, \quad f_s(\alpha) = A_s + (B_s - A_s)\cdot \alpha^q \xrightarrow[q\to\infty]{} A_s \tag{13}$$

- For the pro-core family, we have:

$$\text{For } \alpha \neq 1, \quad f_n(\alpha) = B_n + (A_n - B_n)\cdot (1-\alpha)^q \xrightarrow[q\to\infty]{} B_n \tag{14}$$

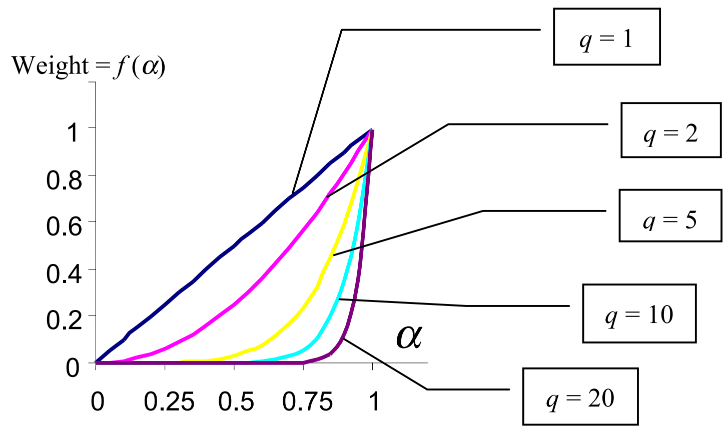
Using these functions for any $q$ we always have that the weights of the core and of the support are fixed by the parameters ($A_i$ and $B_i$) and that they do not change with the variation of $q$.

The pro-support and the pro-core families generalize the functions introduced by Yager and Filev. In fact, we obtain the increasing family with a pro-support function that has the weights $A_S$=0 and $B_S$=1. In a similar way we obtain a decreasing family using a pro-core with the weights $A_C$ =1 and $B_C$ =0. However the analogy between pro-support and increasing family and the pro-core and decreasing family is much more complex, because the increasing or decreasing nature of the pro-support and pro-core is a direct consequence of the relative position of the weights $A_i$ and $B_i$ . For example we have an increasing pro-core function if we have $A_C < B_C$. This is actually the case in the Fig. 10. Obviously we do not have the decreasing family of Yager and Filev, but it somehow reminds their increasing family (Fig. 8). In fact, we have for $q$=1 the same weight distribution. But the behavior of the functions with respect to the variation of $q$ is completely opposite. We obtain by this reasoning two complementary families to the functions introduced by Yager and Filev. In this way we get four different families that we study in the next.

### 5.1. *Four remarkable families*

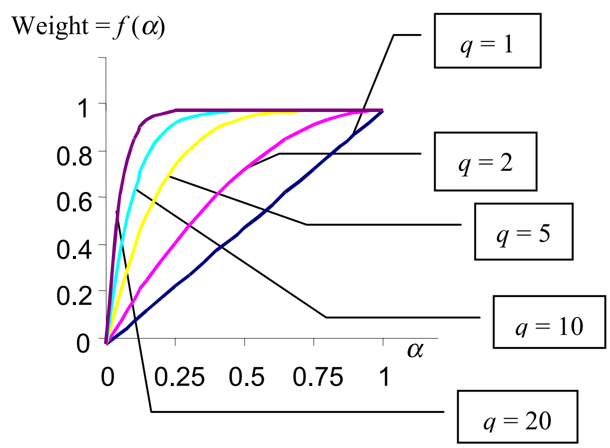
Before studying and giving an intuitive interpretation to the four families of weighting functions, we should keep in mind that what we study (i.e. the function (2)) is nothing else than a defuzzification method [7]. This operation is central in the fuzzy ranking methods, but also of use in fuzzy system modeling, in fuzzy control, in fuzzy decision making and in fuzzy flexible querying (see [8]). Since we focus on a process where we go from fuzzy data to a real number (defuzzification), we are essentially taking into account the different levels of uncertainty with different importance weights. We do that, by associating to with α-level a weight: $f(\alpha)$, indicating its importance weight. In this perspective the selection of the function $f$ corresponds to the choice of a global attitude.

Let us now study the four special families of functions: the Yager and Filev pair and the complementary functions we just introduced.

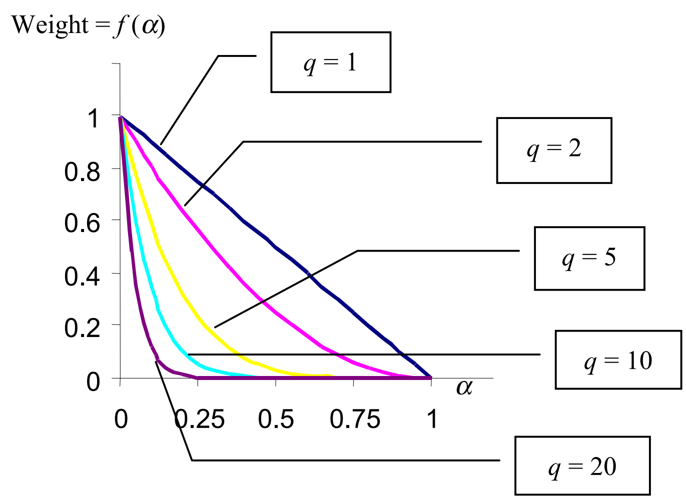



**Increasing pro-support** ($A_S$=0 ; $B_S$=1) **:** $inc_S(\alpha) = \alpha^q$

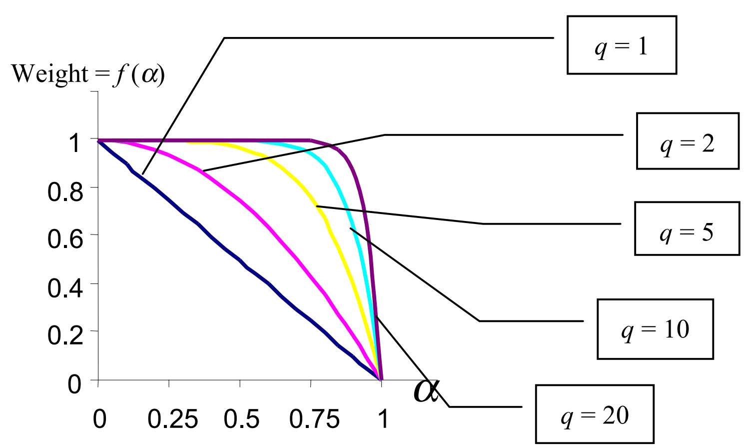
(= increasing function of Yager and Filev)




**Increasing pro-core** ($A_C$ =0 ; $B_C$ =1) : $inc_N(\alpha) = 1 - (1-\alpha)^q$




**Decreasing pro-support** ($A_S$=1 ; $B_S$=0): $dec_S(\alpha) = 1 - \alpha^q$




**Decreasing pro-core** ($A_C$=1 ; $B_C$=0): $dec_C(\alpha) = (1 - \alpha)^q$

(= Yager's and Filev's decreasing family).

## 6. The consequences in the ranking process

We already said that it may be interesting to ponder the different uncertainty levels during the process that transforms a fuzzy set into a crisp value. Here we are going to show a concrete example of why it is interesting in the case of ranking.
What is usually reproached to the ranking methods based on defuzzification is the fact that very imprecise and precise values may be ranked in the same way. This problem is partially solved by the introduction of $\alpha$-weights in the defuzzification (2). Like this, we can for example privilege the certain $\alpha$-levels and doing this rank taking more into account the precise part. But, for example, two symmetric fuzzy numbers centered on the same point will have the same defuzzification value and will not been distinguished even if their precision is not the same. But we think that this result is not against the intuition.
Another critic done to this kind of ranking methods is the brittleness of the ranking. In fact this is the case for defuzzification like mean of maxima, where the expected value is sensitive to a single information that dominates the set. The expected value tend to jump from one frame to the next as the shape of the fuzzy region changes. This problem is solved since we do not only take into account the dominating value (the maxima), but all the $\alpha$-levels. Like this the defuzzified value tends to move smoothly around the output fuzzy region.
Let us now take a look at the problem studied at the beginning of this paper. We show that the final order depends on the choice of the weight function. And it is here were the fact of having different attitudes will be interesting. For example if we are in the particular case were 2 numbers get the same valuation, then knowing the attitude and changing the value of $q$ we will obtain a ranking. In other words, we do not only have an optimistic or pessimistic position with respect to the different certainty levels, but we also have an general attitude that will help us in case of indecision.
Pushing this reasoning further we may think about changing the value of $q$ in order to see if the obtained ranking is stable for our general attitude. This recalls an usual problem that appears with the used defuzzification techniques for ranking methods. In fact, we project the fuzzy number to a real scale which gives us a total pre-order, but two real numbers may be very close to each other and still be strictly smaller (or bigger). This will give a fragile order, because a small modification will change the defuzzified value which will change the order, even if in our case the defuzzified value will only change slightly. Actually, the problem comes from the precision of the real scale. So, we may solve this problem for instance by reducing the number of decimals. This leads to a bigger number of "equal" fuzzy numbers, that we can discriminate by changing the value of $q$.

## 7. Comparison with other defuzzification methods

This study would not be complete without comparing the presented defuzzification method with other existing methods :

- **(COG):** The center of gravity [9] (also called centroid method) is the most common and physically attractive of all the defuzzification methods. The defuzzified value is nothing else than the center of gravity of the represented fuzzy set. Mathematically the center of gravity of *F* is:

$$COG(F) = \frac{\int z \cdot F(z) \cdot dz}{\int F(z) \cdot dz} \tag{15}$$

  Already by looking at the equation we see a certain analogy between this method (15) and ours (2). In both cases we take into account the whole fuzzy number by doing the integral, but here we integrate over the values (i.e. $z$) and for the α-weights we integrate over the α-levels. The fact of taking into account all α-cuts makes the valuation method stable (continuous) [13]. In other words, a small change in the input should not produce a large change in the output.

- **(MOM):** The mean of maxima method [10] (also called the mode method) is often used because of its light computational complexity. The valuation is obtained as an average of the elements which reach the maximal grade in *F*, that is in the normalized case,

$$MOM(F) = Average(F_1) \tag{16}$$

  The α-weight method is nothing else than the generalization of this method to all α-cuts. Actually, the MOM is the limit case where all weights equal zero besides for α=1 (the core). This is the case for the limit ($q \rightarrow \infty$) of the increasing pro-support family. As we already said the MOM is not very stable since the defuzzified value tends to jump from one frame to the next as the shape of the fuzzy region changes. This problem is solved because we take into account all the α-levels and only the dominating value (the maxima).

- **(COM)**: The center of maxima method [11] is a simplified version of the mean of maxima method. Instead of taking all the elements of the α-cut which give the maximal grade, we take the smallest and the largest and we take the middle as valuation. In the case of convex fuzzy sets these two methods are equivalent. This remark may be useful in order to reduce the complexity of the formulation (2).

- **(HM):** The height method [10] (also called the moment method, fuzzy mean method or weighted average method) is considered as a defuzzification method for a simplified fuzzy reasoning method [12] whose consequent part is not a fuzzy set *F* but a real number $z_i$. This simplified method of reasoning is easy to calculate and therefore it is the most widely used as the fuzzy inference method for fuzzy controls and fuzzy-neuro techniques. Here we obtain the valuation as the weighted average of the points $z_i$ with the heights $h_i$.

$$HM\,(z_1,\ldots,z_n) = \frac{\sum_{i=1}^{n} z_i \cdot h_i}{\sum_{i=1}^{n} h_i} \tag{17}$$

The main advantage of this method is the fact that it takes into account the α-levels as weights, which is basically what we do. If we transform (17) from its discrete space to a continuous one we observe that this is nothing else than the defuzzification (2) with the function $f(\alpha)=\alpha$ (pro-support increasing for $q$=1). In other words the α-weight method is a generalization of the height method.

We can now inverse the reasoning and use the results of this study and apply them to a discrete space obtaining like this a height method that takes into account the attitude to be used.

## 8. Conclusions

In this paper we studied the ranking process based on the use of Yager and Filev valuations methods. We tried to understand on simple examples the interaction between valuation family, parameters and ranking result. We began by explaining the valuation method. Then we analyzed and interpreted the valuation of a trapezoidal fuzzy number, what enabled us to discover an interesting property. We also saw how different families can give different orderings for the same collection of triangular fuzzy numbers and that inside a same parameterized family we can invert the ordering of two fuzzy numbers only by changing the value of the parameter. Before finalizing general interpretation of the valuation method. The analysis helped us to find out that some interesting properties were not available using the Yager and Filev functions. For instance the weights of the core and of the support where fixed to the choice of the family. We propose a way to relax this constraint, which allows to set these two weights and keeps them for any variation of $q$. We studied the behavior of this function and discovered that they have the tendency to push their weights (when $q$ increases) to the weight of the core (for the pro-core) and to the weight of the support (for the pro-support). The study of the relaxed functions revealed two complementary families to the Yager and Filev increasing and decreasing functions. We obtained a group of four families that represents an assortment of behaviors. The fact of having these different attitudes enable us to solve a group of major problems related to the ranking using a valuation method, stressed at the beginning of this paper. Since our study is for obvious reasons focused on the valuation (defuzzification) process, we finish by comparing the introduced method to other existing standard methods.

## References

1. Baas, S.M. and Kwakernaak, H., Rating and ranking multiple-aspect alternatives using fuzzy sets, in *Automatica*, 13 , 47-58, 1977.
2. Bortolan, G. and Degani, R., A review of some methods for ranking fuzzy subsets, in *Fuzzy Sets and Systems*, 15, 1-19, 1985.
3. Facchinetti, G., Ricci, R.G. and Muzzioli, S., Note on ranking fuzzy triangular numbers, in *International Journal of Intelligent Systems*, 13, 613-622, 1998.
4. Yager, R.R., A procedure for ordering fuzzy subsets of the unit interval, in *Information Sciences*, 24 , 143-161, 1981.
5. Yager, R.R., On ranking fuzzy numbers using valuations, in *International Journal of Intelligent Systems,* 14, 1249-1268, 1999.
6. Yager, R.R. and Filev D.P., On the instantiation of possibility distributions, in *Technical Report* # MII-1817, Machine Intelligence Institute, Iona College, New Rochelle, NY, 1998.
7. Yager, R.R. and Filev D.P., On the issue of deffuzzification and selection based on a fuzzy set, in *Fuzzy Sets and Systems*, 55, 255-272, 1993.
8. Yager, R.R. and Filev D.P., Essentials of fuzzy modeling and control, *John Wiley: New York*, 1994.
9. Ostergaard J.J., Fuzzy logic control of a heat exchanger process, *Danish technical University Report 7601*, 1976.
10. Brae M. and Rutherford D.A., Fuzzy Relations in a control setting, *Kybernetes 7*, 185-188, 1978.
11. Mamdani E.H., Applications of fuzzy algorithms for control of a simple dynamic plant, *Proc. IEEE 121*, 1585-1588, 1974.
12. Maeda M. and Murakami S., An automobile tracking control with a fuzzy logic, *Proc. 3rd Fuzzy Systems Symp*., Osaka, 61-66, 1987.
13. Mizumoto M., Improvement methods of fuzzy controls, *Proc. 3rd Int. Fuzzy Systems Asoc. Conf., IFSA'89*, Seatle, WA, 1989 pp. 60-62.