

1 A viability approach to control food processes : Application to
2 a Camembert cheese ripening process.

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8 **Abstract**

This paper addresses the issue of studying the viability theory, developed for represents a deep comprehension of the behavioral space in control applications. The aim is to identify the whole set of viable trajectories for a given process. It focuses on the preservation of some specific properties of the system (constraints in the state space). On the basis of this set and a geometric study, a set of actions is identified and robustness is discussed. The proposed framework was adapted to a Camembert ripening model to identify the subset of the space state where almost one evolution starting in the subset remains indefinitely inside of the domain of some viability constraints, that makes it possible to reach a predefined quality target. The results were applied at the pilot scale and are discussed in this paper. The cheese ripening process was shortened by four days without significant changes in the micro-organisms kinetics.

9 *Keywords:* knowledge integration, viability theory, food processing, control, cheese
10 ripening

11 **Nomenclature**

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t	time	(s)
m	cheese mass	(kg)
T_s	cheese surface temperature	(Kelvin)
r_{o_2}	dioxygen consumption rate	$(mol.m^{-2}.s^{-1})$
r_{co_2}	carbon dioxide consumption rate	$(mol.m^{-2}.s^{-1})$
rh	ripening room relative humidity	(%)
T_∞	ripening room temperature	(Kelvin)
w_{o_2}	dioxygen molar mass	$(kg.mol^{-1})$
¹² w_{co_2}	carbon dioxide molar mass	$(kg.mol^{-1})$
s	cheese surface	(m^2)
S	a set of trajectories	
k	the set of constraints	
C	the target to be reached	
x	the vector space of state variables	
SRT	Standard ripening trajectory	
T	the finite time where the target is	days
TVA	Viable ripening trajectory	

¹³ 1. Introduction

¹⁴ The cheese ripening process, such as the one used for Camembert, is considered to be a
¹⁵ complex system. Numerous interactions take place at different levels of scale, from micro-
¹⁶ scopic to macroscopic level, over time. To enhance camembert ripening control, numerous
¹⁷ studies have been carried out in the food sciences, but there is still lack of knowledge.
¹⁸ Despite the number of experimental databases collected, they remain incomplete, and it

19 is obviously impossible to carry out all of the variable combinations through experimental
20 trials because of the time necessary (41 days per trial). However, models have been devel-
21 oped to help us to more effectively understand such complex processes. Cheese processing
22 has been modelled by means of mechanistic models (Riahi et al., 2007), the partial least
23 square method (Cabezas et al., 2006), neuronal methods (Jimenez-Marquez et al., 2003),
24 dynamic Bayesian networks (Baudrit et al., 2008), genetic algorithms (Barriere et al.,
25 2008), stochastic models (Aziza et al., 2006), finite element methods (Bona et al., 2007)
26 and the fuzzy symbolic approach (Perrot, 2004, Ioannou et al., 2003). Simulations can
27 be performed with these models to investigate food processes and to better understand
28 them.

29 The aim of this study was to adapt a viability approach, for control purposes. For
30 study the dynamics of the process with a viability theory (VT) point of view (Aubin
31 et al., 2005), the variables and constraints are characterized by the geometry that its
32 generates in the state space of the model, then the space is classified to identify, for
33 example, the viability kernel : the subset of the space where almost one evolution starting
34 in the subset remains indefinitely inside of the domain of some (viability) constraints. A
35 fundamental difference between VT and classic control engineering, is that VT represents
36 a deep comprehension of the behavioral space, replacing the update procedure from
37 single-valued maps to set-valued maps. For the end user, its knowledge offer a freedom
38 of choice to incorporate new criteria in the decision process. For the decision support
39 system, VT offer an unique oportunity to connect the set structure of the model with
40 an evolutionary optimization mechanism. In control and optimization, the dimension of
41 the problem structure is the first bottleneck for problem solving, it generally define the
42 limits of the application because the curse of dimensionality (CoD). With the benefits

43 of distributed computing environments, it is possible to avoid the CoD without loss
44 of fundamental characteristics of the model (Reuillon et al., 2008), as is required for
45 food processing. This theory has been applied to ecological problems by Bonneuil and
46 Mullers (1997). It was also applied to the renewable resource domain, for example, to
47 the viability of trophic interactions in a marine ecosystem (Chapel et al., 2008) or to
48 the restoration cost of a eutrophic lake (Martin, 2004). Other applications can also be
49 found in the areas of finance (Bonneuil, 2004), highway traffic fluxes (Aubin et al., 2005)
50 and sociology Bonneuil (2000). This is the first time that the viability theory has been
51 applied to food processes. It is applied on the cheese ripening process. It relies on a
52 mathematical development coupling the viability theory developed by Aubin (1991), a
53 high performance computing and an empirical robustness evaluation of the whole viable
54 trajectories extract from expertise handling.

55 The work is presented in **three parts**. **The first part** is dedicated to the theoretical
56 framework of the viability theory and its adaptation to the problem of cheese ripening.
57 The main concepts of the viability theory are defined in Section 2.1 .The adaptation of
58 the concept to the cheese ripening process is developed in the **second part** 2.2. This work
59 also aims at selecting robust process actions from among the set of viable controls for
60 decision help purposes. **The third part** presents an empirical approach of the problem. It
61 is defined in Section 2.4. The viability set and the robust trajectory results are presented
62 in Section 3. In this section, we also describe the test of one of these trajectories during
63 an experimental trial, in comparison to a ripening processed under standard conditions.

64 **2. Material and Methods**

65 *2.1. The viability theory*

66 The viability theory of Aubin (1991) aims at controlling dynamical systems that focus
67 on the preservation of certain specific properties of the system (constraints in the state
68 space).

69 Let $X \subset \mathbb{R}^n$ be the state space of the system. This system state evolves over time
70 $x(\cdot) : t \rightarrow x(t) \in X$ for $t \in R_+ := [0, +\infty[$. We assume that its evolution depends on the
71 state of the system as well as controls. It is governed by a control dynamical system :

$$\begin{cases} x'(t) = f(x(t), u(t)) & (\text{action}) \\ u(t) \in U(x(t)) & (\text{retroaction}) \end{cases} \quad (1)$$

72 where the available controls u at time t belong to the set $U(x(t)) \subset \mathbb{R}^p$. A solution
73 for this system is a trajectory $t \rightarrow x(t)$ so that a measurable control function $t \rightarrow u(t)$
74 exists so that conditions (1) are satisfied for almost all t .

75 Viability constraints are described by a closed subset $K \subset X$ of the state space. They
76 describe the viability of the system since the state of the system is no longer viable
77 outside of K .

78 *2.1.1. Viability kernel*

79 The general definition of the basis of the viability theory is the viability kernel, referred
80 to as $Viab_{f,U}(K)$, which contains all states from which at least one control function $u(t)$
81 exists so that the state of the system $x(t)$ remains in K for t in $[0, T]$. We recall that
82 $S_{f,U}(x)$ is the set of all trajectories governed by the controlled dynamical system (1)
83 starting from x . The viability kernel is then defined by Equation 2:

$$Viab_{f,U}(K) := \{x \in K \mid \exists x(\cdot) \in S_{f,U}(x), \forall t \in [0, T], x(t) \in K\} \quad (2)$$

84 This viability kernel also determines the set of controls that would prevent the system
 85 from violating the state constraints. The particular case of the capture basin is to find
 86 trajectories remaining in the constraint domain that reach a target C within a finite
 87 time. This is a variant of the viability problem (Equation 3) known as capture basin
 88 $Capt_{f,U}(K, C)$.

$$Capt_{f,U}(K, C) = \{x \in K \mid \exists x(\cdot) \in S_{f,U}(x), \exists t^* > 0, x(t^*) \in C, \forall t \in [0, t^*], x(t) \in K\} \quad (3)$$

89 t^* is the time at which the target is reached. The trajectory $x(\cdot)$ must remain in the
 90 constraint set K before reaching the target C . For our application, the target C is the
 91 Camembert characteristic to be reached. For example cheese mass must be at least be of
 92 0.25 kg (defined by the protected designation of origin law).

93 2.2. The Camembert ripening model

94 The evolution of Camembert ripening was considered to be governed by cheese mass
 95 loss dynamics, including microorganism respiration described in Equations (4) and (5)
 96 Helias et al. (2007).

$$\frac{dm}{dt} = s \{w_{o_2} \cdot r_{o_2} - w_{co_2} \cdot r_{co_2} - k [a_w \cdot p_{sv}(T_s) - rh \cdot p_{sv}(T_\infty)]\} \quad (4)$$

$$\frac{dT_S}{dt} = \frac{s}{m.C} \left\{ h(T_\infty - T_s) + \varepsilon\sigma(T_\infty^4 - T_s^4) - \lambda k [a_w \cdot p_{sv}(T_s) - rh \cdot p_{sv}(T_\infty)] + \alpha \frac{r_{o_2} + r_{co_2}}{2} \right\} \quad (5)$$

97 In these equations, t represents the time, m the cheese mass (kg), T_s the tempera-
 98 ture at the cheese surface (*Kelvin*), r_{o_2} the oxygen consumption rate ($mol.m^{-2}.s^{-1}$),
 99 r_{CO_2} the dioxyde production rate ($mol.m^{-2}.s^{-1}$), rh the relative humidity (expressed be-
 100 tween 0 and 1) and T_∞ the temperature in the ripening room (*Kelvin*). The parameters
 101 w_{o_2} and w_{co_2} are molar masses ($kg.mol^{-1}$), s is the cheese surface (m^2), a_w is the cheese
 102 surface water activity (*dimensionless*), p_{sv} is the saturation vapor pressure (Pa), k is
 103 the average water transfer coefficient ($kg.m^{-2}.Pa^{-1}.s^{-1}$), C is the cheese specific heat
 104 ($J.kg^{-1}.K^{-1}$), h is the average convective heat transfer coefficient ($W.m^{-2}.K^{-1}$), ε is the
 105 cheese emissivity (*dimensionless*), σ is the Stefan-Boltzmann constant ($W.m^{-2}.K^{-4}$),
 106 α is the respiration heat for 1 mol of carbon dioxide release ($J.mol^{-1}$) and λ is the la-
 107 tent vaporization heat of water ($J.kg^{-1}$). The control variables considered in this model
 108 are relative humidity and temperature. The state variables are the cheese mass and the
 109 cheese surface temperature. This model was developed and validated on experimental
 110 data sets with a relative error between 1.9-3.2%.

111 In order to be able to use it for simulation, the model was modified so that the gas
 112 composition was no longer measured online but was instead extrapolated from experi-
 113 mental curves of microorganism respiration during ripening at 281 K, 285 K and 289 K
 114 and at 92% relative humidity. The input and output are cheese mass, cheese surface tem-
 115 perature and respiration r_{co_2} (r_{o_2} is deduced from r_{co_2} with the assumption of equimolarity
 116 Helias et al., 2007).

117 This empirical respiration model induces uncertainty in the prediction. The aim was to
118 test the viability theory for this commonly encountered case because model generalisation
119 is rarely perfect.

120 *2.3. Determining the viability kernel for camembert cheese ripening process : algorithm*
121 *and computation*

122 Numerical schemes to solve ‘viability’ or ‘capture’ problems were proposed by Saint-
123 Pierre (1994): for a given time step Δt and a given grid G_h in the state space, the viability
124 kernel algorithm computes a discrete viability kernel that converges to the viability ker-
125 nel $Viab_{f,u}(K)$ when the time step and the grid resolution tend toward 0. This is the
126 approach used in this work, the ripening model was discretised over time using a Euler
127 scheme. Moreover, the state space, the control space, the constraints and the target were
128 discretised on regular grids.

129 *2.3.1. The constraint set*

130 The vector space X consists of three state variables: cheese mass, cheese surface
131 temperature and respiration level (see Section 2.2). The constraints set is a subset of this
132 three dimensional space. The bound values stem from the experimental limits, the legal
133 norms and expert interviews presented in table 1.

134 [Table 1 around here]

135 An additional constraint concerns the state variable of microorganism respiration.
136 The hypothesis proposed is that the evolution of the respiration rate is an indicator of
137 the microorganism growth necessary for Camembert cheese ripening. This hypothesis

138 was developed on the basis of studies by Couriol et al. (2001) and Adour et al. (2002).
139 The respiration rate should increase up to at least $8.10^{-6}mol.m^{-2}.s^{-1}$ during ripening.

140 *2.3.2. Quality target to be reached*

141 First, the cheese surface temperature is fixed between 281 K and 289 K at the end
142 of the ripening process according to experts knowledge in order to easily manipulate
143 the cheeses. The second target dimension to be reached is the Camembert mass. We
144 established a target between [0.25; 0.27] kg. Finally, the third dimension to be taken into
145 account is the microorganism respiration which ensure good cheese sensory properties.
146 It is fixed at the end of ripening between $[6; 13].10^{-6}mol.m^{-2}.s^{-1}$ for a target of r_{CO_2} of
147 $[10, 25]g.m^{-2}.day^{-1}$. The standard time spent in the ripening room is around 12 days
148 before the cheese is wrapped. A first viability kernel was computed with a ripening time
149 of 12 days. The aim was then to evaluate a shorter ripening time. To do this, another
150 viability kernel is calculated for $T= 8$ days.

151 *2.3.3. The controls*

152 Concerning the controls, the ripening room temperature is chosen from between 281
153 K and 289 K by increments of 1°K. The relative humidity is chosen from 84% to 98% by
154 increments of 2% (maximum precision of the sensor). The control change (temperature
155 and/or relative humidity) was limited to a frequency of one per 24 h.

156 *2.3.4. The algorithm used to determine the viability kernel*

157 The viability kernel was calculated from the target (end of ripening) to time 0 (be-
158 ginning of ripening) by means of Algorithm 1. In this algorithm, D_t is the discretised
159 set of the viable state at t , and T is the finite time where the target is reached. The

160 discretised target and the constraints are referred to as C_h and K_h , respectively. The
 161 term, $Succ(x)$, represents the successors of x . $Succ(x)$ is the result $(m_{t+1}, T_{s_{t+1}}, r_{co_2t+1})$
 162 of the Camembert ripening model applied to $x \in K_h$ with position $(m_t, T_{s_t}, r_{co_2t}, t)$. The
 163 viability kernel is built from all of the viable state x at each time interval.

164 Algorithm 1: **Initialization** $D_T \leftarrow C_h$; **Main loop**; For $t := T - 1$ to 1; $D_t \leftarrow$
 165 $\{x \in K_h | Succ(x) \cap D_{t+1} \neq \emptyset\}$; **Return** $\{D_1, D_2, \dots, D_T\}$

166 2.3.5. High performance computing

167 The main difficulty in calculating the viability kernel is the dimension of the space
 168 to be explored. For example, it is necessary to test 4 150 440 points (controls*states)
 169 multiply by 11 days (day 12 = target C) for a ripening time of 12 days. Therefore, 45
 170 654 840 simulations have to be performed with the Camembert ripening model. The
 171 calculation time was estimated at 1.5 months on a single computer. As a result, the
 172 calculation was distributed in a high performance calculation structure, the MIG-cluster
 173 (INRA, Jouy-en-Josas). The viability algorithm was computed with Matlab (The Math-
 174 Works, Inc., MA, USA) and then transferred to Octave¹ free software for the calculation
 175 distribution. The calculation time was reduced to seven days with the 200 CPU (Central
 176 Process Unit) of the MIG-cluster.

177 2.4. Empirical robustness evaluation of the viable trajectories

178 The robustness of each ripening trajectory $x(\cdot)$ is defined and calculated by

$$Rob(x(\cdot)) := \sum_{t=1}^{T-1} \left(\frac{\#Cont_v(x(t))}{\max_{x \in D_t} \#Cont_p(x, t)} \right) \quad (6)$$

1. www.gnu.org/software/octave/

179 ; where $Cont_v(x(t))$ represents the number of viable controls at state $x(t)$ and $Cont_p(x, t)$
180 the number of possible controls at state $x(t)$.

181 2.5. Cheese ripening trials

182 To test the ripening trajectory found with the viability method, Camembert-type soft
183 mould cheeses were manufactured as described by (Leclercq-Perlat et al., 2004) under
184 aseptic conditions in a sterilised 2-m³ cheesemaking chamber (figure 1). During, the pilot
185 trial, several indicators were continuously measure in the ripening chamber : temperature,
186 relative humidity, respiratory activity of the microorganims and cheese mass loss.

187 [Figure 1 around here]

188 The sensory analysis was performed by the sensory analysis company Actilait (Maison
189 du Goût, Rennes) at day 35 after cheesemaking. This day was choosen as a time
190 reference. The cheeses were evaluated on the basis of 26 indicators on a continuous 10-
191 point scale. The sensory panel also assessed cheeses from a dairy company purchased in a
192 supermarket. The aim was to compare the sensory profile of the experimental cheeses to
193 commercial cheeses. Finally, the data analysis was performed with the Matlab software
194 (The MathWorks, Inc., MA, USA). A two-way variance analysis (ANOVA) was carried out
195 separately on each attribute according to the following model: attribute = product +
196 repetition + product × repetition. When significant product differences were observed
197 ($P < 0.05$), product mean intensities were compared using the Tukey-Kramer multiple
198 comparison test.

199 Two ripening trials were performed in this study. One trial was a standard ripen-
200 ing trial within 12 days in the ripening room at 92% relative humidity and 285 K and

201 the cheeses were wrapped and stored at 277 K. This standard ripening trial is the one
202 typically used in dairy industry. The second trial was controlled along the trajectory cal-
203 culated using the viability approach. The cheeses were ripened for 8 days in the ripening
204 room before being wrapped.

205 3. Results

206 We first present the computed viability kernels for two different process times (8 and
207 12 days). We then describe the results reached within pilot experimentations for the
208 trajectory calculated using the viability approach (TVA). Finally results are compared
209 to a standard ripening trajectory (SRT).

210 3.1. Viability kernels

211 Two viability kernels were calculated. The discrete viability kernel corresponding to
212 12 days of ripening is presented in Figure 2. At day 12, the viable states represented
213 correspond to the target C. The kernel is thin at the beginning because the respiration
214 rate is at the 0 level corresponding to the latency phase of microorganisms. At day 1,
215 cheese masses lower than 0.262 kg are not viable and the respiration rate should be at the
216 0 level. The number of viable respiration rates then reaches a maximum in the middle of
217 the process and decreases at the end. Concerning cheese mass, the viable maximal mass
218 obviously decreases. All the cheese surface temperatures between 281 K and 289 K are
219 viable throughout the process.

220 [Figure 2 around here]

221 *3.2. Viable trajectories*

222 Several trajectories selected using the first empirical robustness calculus were found in
223 the 12-day viability kernel. The criteria was those defined by the cheesemakers from the
224 dairy industry: limit the control variation as to reduce operational costs and reduce the
225 initial cheese mass as to reduce the necessary raw material. One efficient viable trajectory
226 was found for a 0.284 kg cheese and a 8-day ripening period. This trajectory was four
227 days shorter than the standard ripening period. The controls for this trajectory (TVA)
228 are presented in Figure 3b and the controls for the nominal one typically used in dairy
229 industry (SRT) is presented in Figure 3a.

230 [Figure 3 around here]

231 The TVA trajectory differs from the classical one. The relative humidity is constant
232 but 2% higher 94% instead of 92% and the temperature control is modified instead of
233 remaining the same.

234 *3.3. Application of the viable ripening trajectory (TVA) on a pilot and comparison to a*
235 *standard one (SRT)*

236 The TVA trajectory was then applied in a pilot. The results for cheese mass loss
237 evolution, microbiological and physicochemical kinetics were compared to those obtained
238 during standard ripening on this pilot. The sensory quality of the manufactured cheeses
239 was also compared to a commercial one.

240 *3.3.1. Cheese mass loss evolution*

241 Figure 4 shows the mass loss measured during the trial TVA compared to the mass
242 loss measured during the standard trial (SRT). The mass loss is 0.034 kg for the robust

243 ripening and 0.054 kg for the standard ripening. The yield for the viable ripening is about
244 89% and the one of the standard ripening is about 85%.

245 [Figure 4 around here]

246 The cheese mass at the end of the robust ripening is within the desired target (0.25 kg-0.27
247 kg).

248 3.3.2. Comparison of microbiological and physicochemical kinetics

249 The respiration rate and microbial activities of viable (TVA) and standard (SRT)
250 ripening processes were also compared. The results are presented in Figure 5 for the
251 respiration rate and microorganisms growth. As projected, the respiration rate began at
252 0, reached a maximum of over $8.10^{-6} mol.m^{-2}.s^{-1}$ and then slowly decreased until the day
253 the cheese was wrapped. The maximum respiration rate began 1 day earlier in theTVA
254 ripening process than in the standard ripening process but is preserved. Concerning the
255 pH, it increases approximately one day earlier in the TVA ripening process than in the
256 standard one. For the microorganisms growth, differences are limited and kinetics trends
257 are similar for The yeast *K. marxianus*, *G. candidum*. Concerning *B.aurantiacum*, growth
258 occurred at the same time for the TVA ripening and for the SRT ripening. However, the
259 level of *B.aurantiacum* was always lower in the case of the viable trajectory.

260 [Figure 5 around here]

261 3.3.3. Comparison with commercial camembert cheeses

262 Cheeses ripened under standard conditions (SRT) and viable conditions (TVA) were
263 assessed by a sensory panel at day 35 and compared to a commercial cheese. The differ-
264 ence between the cheeses was explored with a Tukey-Kramer significance difference test.

265 The results are given in Table 2 and figure 6. The cheese reached with the TVA trajectory
266 was found to be very close to the standard cheese. Only three sensory indicators have
267 revealed significant differences between the cheeses: the core color indicator, the chalky
268 core indicator and the hard texture indicator.

269 [Table 2 around here]

270 [Figure 6 around here]

271 4. Conclusion

272 Thanks to the viability theory framework we were able to compute the set of all viable
273 trajectories that satisfy the manufacturing constraint and to reach the quality target for
274 the ripening process. We evaluated an empirical robustness on these trajectories and
275 choose a trajectory with low operational costs from among the more robuste ones. This
276 trajectory has a 8-day ripening time and an initial mass of 0.284 kg, whereas the standard
277 is 12 days and 0.3 kg. This trajectory was validated on a ripening pilot. The microbial
278 equilibrium was preserved so as the cheese sensory properties. We can then conclude that
279 the trajectory built with the viability theory is realistic. The viability method allowed us
280 to effectively propose a pertinent approach of control for the cheese ripening process. It is
281 CPU time consuming. Nevertheless the real value added of this method, by comparison
282 to a control optimal search, is the possibility to describe the whole viable trajectories.
283 As a consequence we are able to calculate the frontier of the viable set and the distance
284 of each trajectory to this frontier. Further studies will be focus on the development of a
285 geometric analysis of the viability kernel for robustness qualification of each trajectories.

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290 **References**

291 **References**

- 292 Adour, L., Couriol, C., Amrane, A., Prigent, Y., Sep. 2002. Growth of *Geotrichum candidum* and peni-
293 cillium *camemberti* in liquid media in relation with the consumption of carbon and nitrogen sources
294 and the release of ammonia and carbon dioxide. *Enzyme And Microbial Technology* 31 (4), 533–542.
- 295 Aubin, J. P., 1991. Tracking property - a viability approach. *Lecture Notes In Control And Information*
296 *Sciences* 154, 1–15.
- 297 Aubin, J. P., Bayen, A. M., Saint-Pierre, P., 2005. A viability approach to hamilton-jacobi equations:
298 application to concave highway traffic flux functions. 2005 44th IEEE Conference on Decision and
299 Control & European Control Conference, Vols 1-8, 3519–3524.
- 300 Aziza, F., Mettler, E., Daudin, J. J., Sanaa, M., 2006. Stochastic, compartmental, and dynamic modeling
301 of cross-contamination during mechanical smearing of cheeses. *Risk Analysis*: 26 (3) 731-745 26 (3),
302 731–745.
- 303 Barriere, O., Lutton, E., Baudrit, C., Sicard, M., Pinaud, B., Perrot, N., 2008. Modeling human expertise
304 on a cheese ripening industrial process using gp. *Parallel Problem Solving From Nature - Ppsn X*,
305 *Proceedings* 5199, 859–868.
- 306 Baudrit, C., Wuillemin, P. H., Sicard, M., Perrot, N., 2008. A dynamic bayesian network to represent
307 a ripening process of a soft mould cheese. *Knowledge-Based Intelligent Information And Engineering*
308 *Systems, Pt 2, Proceedings* 5178, 265–272.
- 309 Bona, E., da Silva, R. S. S. F., Borsato, D., Silva, L. H. M., Fidelis, D. A. D., Apr. 2007. Multicomponent
310 diffusion modeling and simulation in prato cheese salting using brine at rest: The finite element method
311 approach. *Journal of Food Engineering* 79 (3), 771–778.

312 Bonneuil, N., 2000. Viability in dynamic social networks. *Journal Of Mathematical Sociology* 24 (3),
313 175–192.

314 Bonneuil, N., Mullers, K., Feb. 1997. Viable populations in a prey-predator system. *Journal Of Mathe-*
315 *matical Biology* 35 (3), 261–293.

316 Bonneuil, N. et Saint-Pierre, P., 2004. The hybrid guaranteed capture basin algorithm in economics.
317 *Hybrid Systems: Computation And Control, Proceedings 2993*, 187–202.

318 Cabezas, L., Gonzalez-Vinas, M., Ballesteros, C., Martin-Alvarez, P. J., Feb. 2006. Application of partial
319 least squares regression to predict sensory attributes of artisanal and industrial manchego cheeses.
320 *European Food Research and Technology* 222 (3-4), 223–228.

321 Chapel, L., Defuant, G., Martin, S., Mullon, C., Mar. 2008. Defining yield policies in a viability approach.
322 *Ecological Modelling* 212 (1-2), 10–15.

323 Couriol, C., Amrane, A., Prigent, Y., Jun. 2001. A new model for the reconstruction of biomass history
324 from carbon dioxide emission during batch cultivation of *geotrichum candidum*. *Journal of Bioscience*
325 *and Bioengineering* 91 (6), 570–575.

326 Helias, A., Mirade, P. S., Corrieu, G., 2007. Modeling of camembert-type cheese mass loss in a ripening
327 chamber: Main biological and physical phenomena. *Journal of Dairy Science* 90, 5324–5333.

328 Ioannou, I., Perrot, N., Mauris, G., Trystram, G., 2003. Experimental analysis of sensory measurement
329 imperfection impact for a cheese ripening fuzzy model. *Fuzzy Sets and Systems - Ifsa 2003, Proceedings*
330 2715, 595–602.

331 Jimenez-Marquez, S. A., Lacroix, C., Thibault, J., May 2003. Impact of modeling parameters on the
332 prediction of cheese moisture using neural networks. *Computers & Chemical Engineering* 27 (5),
333 631–646.

334 Leclercq-Perlat, M. N., Buono, F., Lambert, D., Latrille, E., Spinnler, H. E., Corrieu, G., 2004. Controlled
335 production of camembert-type cheeses. part i: Microbiological and physicochemical evolutions. *Journal*
336 *of Dairy Research* 71 (3), 346–354.

337 Martin, S., Dec. 2004. The cost of restoration as a way of defining resilience: a viability approach applied
338 to a model of lake eutrophication. *Ecology And Society* 9 (2), 8.

339 Perrot, N., 2004. De la maitrise des procédés alimentaires par intégration de l'expertise humaine.
340 Le formalisme de la théorie des ensembles flous comme support. HDR. Université Blaise Pascal,
341 Clermont-Ferrand, France.

- 342 Reuillon, R., Hill, D. R. C., El Bitar, Z., Breton, V., 2008. Rigorous distribution of stochastic simulations
343 using the distme toolkit. IEEE TRANSACTIONS ON NUCLEAR SCIENCE 55, 595–603.
- 344 Riahi, M. H., Trelea, I. C., Picque, D., Leclercq-Perlat, M. N., Helias, A., Corrieu, G., 2007. Model de-
345 scribing debaryomyces hansenii growth and substrate consumption during a smear soft cheese deacid-
346 ification and ripening. Journal of Dairy Science 90 (5), 2525–2537.
- 347 Saint-Pierre, P., 1994. Approximation of the viability kernel. Applied Mathematics and Optimization
348 29, 187–209.

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Table 1: The vector space for the three state variables of the camembert ripening model

	Unit	Min	Max	Steps
Mass	g	250	310	1
Cheese surf temperature	kelvin	281	289	1
Respiration	$gCO_2.m^{-2}.day^{-1}$	0	55	1

370

371

Table 2: Multicomparison test of sensory results for cheeses with robust ripening (Robcheese), cheeses with standard ripening (Standardcheese) and cheeses from a dairy industry (Commercialcheese) based on Tuckey-Kramer significant difference.

	RobCheese	StandardCheese	CommercialCheese
Rind colour	a	a	b
Rind regularity	a	a	b
Core colour	a	b	ab
Chalky core	a	b	ab
Runny core	a	a	b
Aperture quantity	a	a	b
Ammoniac odour	a	a	b
Soft texture	a	a	b
Hard texture	a	b	ab
Sticky texture	a	a	b
Creamy texture	a	a	b
Animal aroma	a	a	b
Ammoniac aroma	a	a	b

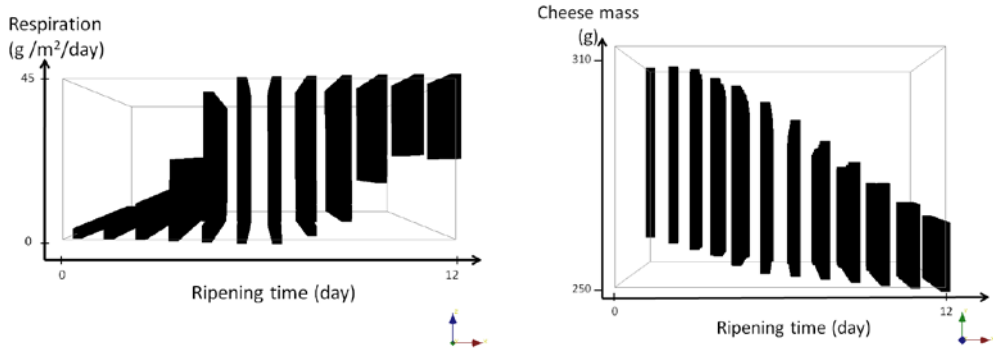


Figure 2

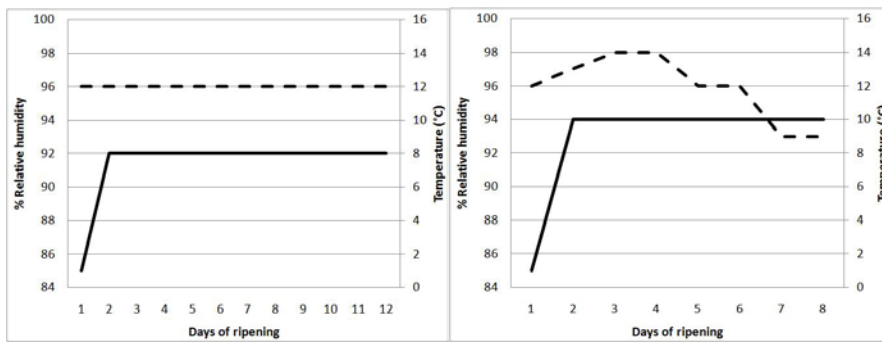


Figure 3

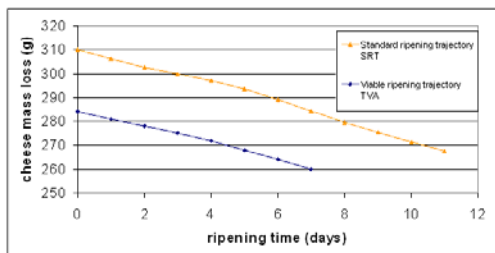


Figure 4 *****

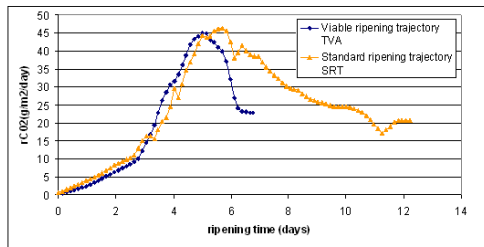
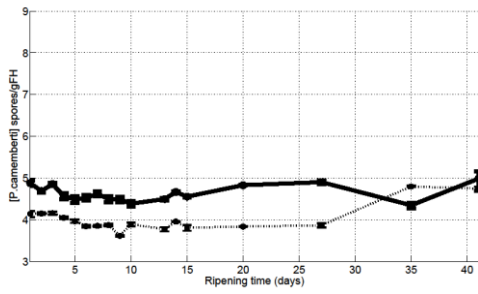
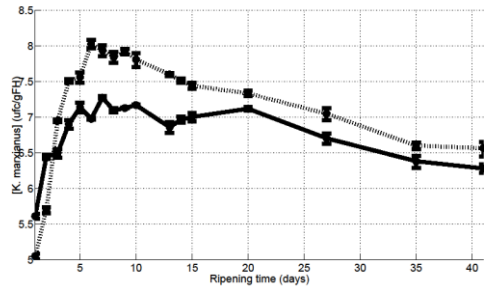
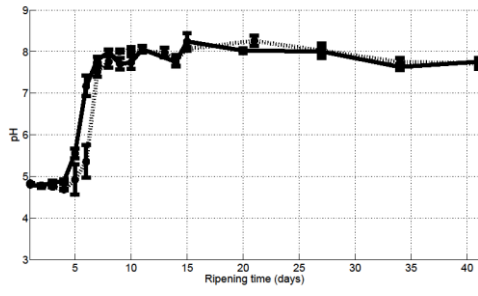


Figure 5 a,b,d et f

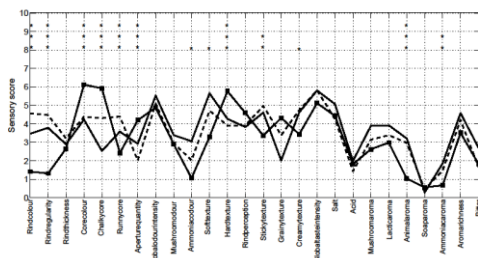


Figure 6 ****